

Let P be the point $(-2, 3, -5)$. Let Q be the point $(-8, -1, -3)$.

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Let R be the point such that $\overrightarrow{PR} = \langle 1, 3, 2 \rangle$.

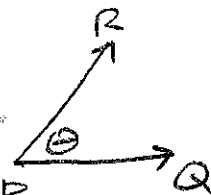
- [a] Find the co-ordinates of R .

$$(-2+1, 3+3, -5+2) = (-1, 6, -3)$$

- [b] In the triangle ΔPQR , find the measure of angle $\angle RPQ$.

$$\overrightarrow{PQ} = \langle -8 - (-2), -1 - 3, -3 - (-5) \rangle = \langle -6, -4, 2 \rangle$$

$$\cos^{-1} \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{\|\overrightarrow{PQ}\| \|\overrightarrow{PR}\|} = \cos^{-1} \frac{-6 - 12 + 4}{\sqrt{56} \sqrt{14}} = \cos^{-1} \frac{-14}{2(14)} = \cos^{-1} -\frac{1}{2} = \frac{2\pi}{3}$$



- [c] Write $\overrightarrow{PR}$ as the sum of two orthogonal vectors, one of which is the projection of $\overrightarrow{PR}$ onto $\overrightarrow{PQ}$.

$$\frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{\overrightarrow{PQ} \cdot \overrightarrow{PQ}} \overrightarrow{PQ} = \frac{-14}{56} \langle -6, -4, 2 \rangle = -\frac{1}{4} \langle -6, -4, 2 \rangle = \langle \frac{3}{2}, 1, -\frac{1}{2} \rangle$$

$$\langle 1, 3, 2 \rangle = \langle \frac{3}{2}, 1, -\frac{1}{2} \rangle + \langle -\frac{1}{2}, 2, \frac{5}{2} \rangle$$

- [d] Find the area of the triangle ΔPQR .

$$\begin{aligned} \overrightarrow{PQ} \times \overrightarrow{PR} &= \langle -6, -4, 2 \rangle \times \langle 1, 3, 2 \rangle \\ &= \langle -8 - 6, -(-12 - 2), -18 + 4 \rangle \\ &= \langle -14, 14, -14 \rangle \end{aligned}$$

$$\frac{1}{2} \|\overrightarrow{PQ} \times \overrightarrow{PR}\| = \frac{1}{2} (14) \|\langle -1, 1, -1 \rangle\| = 7\sqrt{3}$$

- [e] Find the equation of the plane passing through P, Q and R . Write your final answer in general form $Ax + By + Cz + D = 0$.

$$-14(x+2) + 14(y-3) - 14(z+5) = 0$$

$$\text{or } (x+2) - (y-3) + (z+5) = 0$$

$$x - y + z + 10 = 0$$

Let $\vec{r}$ be the vector with magnitude 4 and direction angle $\frac{\pi}{6}$. Let $\vec{s} = \langle -4\sqrt{3}, 4 \rangle$.

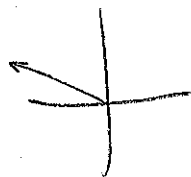
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[a] Find the component form of $\vec{r}$.

$$\langle 4 \cos \frac{\pi}{6}, 4 \sin \frac{\pi}{6} \rangle = \langle 2\sqrt{3}, 2 \rangle$$

[b] Find the direction angle of $\vec{s}$.

$$\tan^{-1} \frac{4}{-4\sqrt{3}} = \tan^{-1} -\frac{\sqrt{3}}{3} = -\frac{\pi}{6} \text{ in } Q_4$$


$$\pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

[c] If $\vec{r}$ represents a force, and $\vec{s}$ is the movement of an object that the force is applied to, find the work done.

$$\langle 2\sqrt{3}, 2 \rangle \cdot \langle -4\sqrt{3}, 4 \rangle = -8(3) + 8 = -16$$

If $\langle 2, a, -12 \rangle$ is parallel to $\langle b, -8, 9 \rangle$, find the values of a and b .

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$$\langle 2, a, -12 \rangle = k \langle b, -8, 9 \rangle$$

$$2 = kb$$

$$a = -8k$$

$$-12 = 9k \rightarrow k = -\frac{4}{3}$$

$$a = -8\left(-\frac{4}{3}\right) = \frac{32}{3}$$

$$2 = -\frac{4}{3}b \rightarrow b = -\frac{3}{2}$$

Let $\vec{b} = 3\vec{i} - 6\vec{j} - 2\vec{k}$ and $\vec{c} = -2\vec{j} + \vec{k}$.

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Let L be the point $(5, -3, 4)$.

[a] Write $3\vec{c} - 2\vec{b}$ in component form.

$$3\langle 0, -2, 1 \rangle - 2\langle 3, -6, -2 \rangle = \langle 0, -6, 3 \rangle - \langle 6, -12, -4 \rangle \\ = \langle -6, 6, 7 \rangle$$

[b] Find a vector of magnitude 3 in the same direction as $\vec{b}$.

$$3\left(\frac{1}{\|\vec{b}\|}\right)\vec{b} = 3\left(\frac{1}{\sqrt{49}}\right)\langle 3, -6, -2 \rangle = \frac{3}{7}\langle 3, -6, -2 \rangle \\ = \left\langle \frac{9}{7}, -\frac{18}{7}, -\frac{6}{7} \right\rangle$$

[c] Find a unit vector perpendicular to both $\vec{b}$ and $\vec{c}$.

$$\vec{b} \times \vec{c} = \langle 3, -6, -2 \rangle \times \langle 0, -2, 1 \rangle \\ = \langle -6-4, 0-3, -6-0 \rangle \\ = \langle -10, -3, -6 \rangle$$

$$\frac{1}{\|\vec{b} \times \vec{c}\|}(\vec{b} \times \vec{c}) = \frac{1}{\sqrt{145}}\langle -10, -3, -6 \rangle = \left\langle \frac{-10}{\sqrt{145}}, \frac{-3}{\sqrt{145}}, \frac{-6}{\sqrt{145}} \right\rangle$$

[d] Find the equation of the plane parallel to both $\vec{b}$ and $\vec{c}$, and passing through point L .
Write your final answer in general form $Ax + By + Cz + D = 0$.

$$-10(x-5) - 3(y+3) - 6(z-4) = 0 \\ \text{or } 10(x-5) + 3(y+3) + 6(z-4) = 0 \\ 10x + 3y + 6z - 65 = 0$$

[e] Let ℓ_1 be the line with parametric equation $x = 3t - 5$, $y = 6 - 2t$, $z = t + 8$.
Find the symmetric equation of the line parallel to ℓ_1 and passing through L .

$$\frac{x-5}{3} = \frac{y+3}{-2} = \frac{z-4}{1}$$

$$\text{or } \frac{x-5}{3} = -\frac{y+3}{2} = z-4$$